

A Robust Test for Equality of Variances of k Normal Populations

K.Aruna Rao

*University of Botswana, Statistics Department
Gaborone, Botswana
raoka@noka.ub.bw*

Meena Badade

*University of Botswana, Statistics Department
Gaborone, Botswana
badademn@noka.ub.bw*

This paper introduces a test based on modified version of Gini's mean difference for testing equality of scale parameters. If k independent samples of sizes n_i ($i = 1, 2, \dots, k$) are taken from different normal populations, the hypothesis of equality of population variances ($H_0 : \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2$) can be tested using Bartlett's test. However, it is i) sensitive to non-normality; and ii) affected in the presence of outliers. To cover these inadequacies, a new test based on Gini's mean difference is proposed. The test is based on the difference between logarithm of arithmetic and geometric mean of estimators of population variances. The test statistic can be defined as

$$(1) \quad T = \sum n_i \log G_w^2 - \sum n_i \log G_i^2, \text{ where}$$

$$G_i = \frac{\sqrt{\pi}}{n_i(n_i - 1)} \sum_{j < l} (x_{i(j)} - x_{i(l)}), \quad G_w^2 = \frac{\sum n_i G_i^2}{\sum n_i}, \quad i = 1, 2, \dots, n_i.$$

Under H_0 , the distribution of T can be approximated by linear combinations of independent χ_1^2 variates. A further modification uses test statistic $T_1 = \frac{T}{\bar{\lambda}}$ where $\bar{\lambda}$ is the mean of respective eigenvalues appearing in the linear combinations. T_1 is approximately distributed as χ_k^2 . The proposed test was compared to Bartlett's with respect to the following aspects:

1. Size - The size of the two tests were computed through 1000 simulations using either 4 or 8 groups and average group size = 6, 10, 14, 18 and 22. The simulated values were chosen from normal distribution with mean and variance 100. Four different cases were studied: i) balanced - all group sizes equal; ii) imbalanced 1 - difference of 1 or 2 between successive group sizes; iii) imbalanced 2 - one group size large, others small and almost equal; iv) imbalanced 3 - one group size small, others large and almost equal. Case (ii) was moderately imbalanced, whereas case (iii) and (iv) were variations of extreme imbalance. The total number of configurations was 40 at each l.o.s.(α). The three l.o.s. used in this exercise were 1%, 5% and 10%.

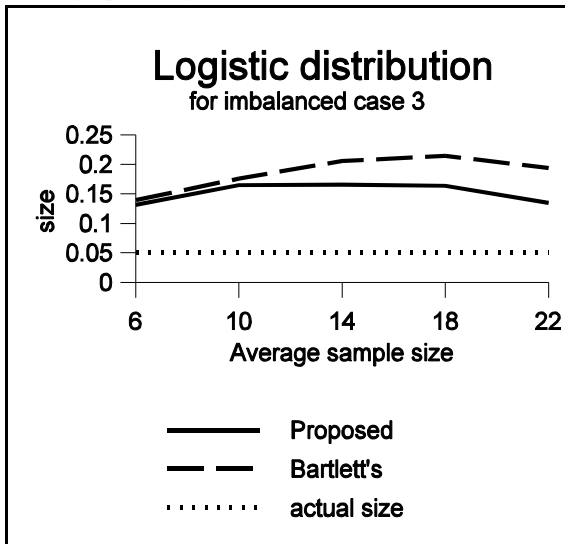
For Bartlett's test, the simulated sizes were in close agreement with corresponding l.o.s. But for proposed test, the difference between the two was remarkable which narrowed down with increasing sample size. Hence further comparisons were made using percentile values obtained through simulation and not χ^2 table values.

2. Power - The power of the two tests was compared for two cases: A) increasing variance for increasing group size; B) decreasing variance for increasing group size. Both tests were found to be at par. However, Bartlett's test had a slight edge over the other for case (A) and proposed test looked more powerful for case (B). (See Table 1.)

Table 1. Power difference between Bartlett's and proposed test at 5% l.o.s. for case (B)

Bartlett's power -Proposed test's power	Frequency	Comment
less than -0.01	18	proposed test more powerful
-0.01 to 0.01	15	both tests in close agreement
more than 0.01	7	Bartlett's test more powerful

Average power for Bartlett's test = 0.5414



Kuva 1 size for proposed & Bartlett's test for Logistic distribution for imbalanced case 3, 4 groups, l.o.s. = 5%

3. Parent populations non-normal - Two non-normal populations were chosen: i) lognormal ii) logistic. For both, mean and variance were also chosen to be 100. An index of agreement calculated as $\frac{100(\alpha - size(actual))}{\alpha}$ was used. For both

distributions, level of agreement between actual size and α increased with increasing α . For lognormal distribution, both tests seemed to be at par with non-significant variations. However, for logistic distribution, proposed test showed more robustness over Bartlett's. (See Figure 1.)

4. Presence of outlier - An outlier with A) small mean ($100 + \sigma$), B) large mean ($100 + 3\sigma$), C) small variance ($2.25\sigma^2$) or D) large variance ($9\sigma^2$) was inserted in either the smallest or the largest group and both tests were studied for robustness. The degree of robustness was found to increase with increasing α . For case (A) and (C), both tests were found to be nearly at par with Bartlett's test

showing slight edge for case (C). For case (B) and (D), proposed test appeared more robust than Bartlett's. (See Table 2.)

Table 2. Index of agreement between actual size and α in presence of outlier with variance $9\sigma^2$

	Index of agreement	Test	$\alpha = 1\%$	$\alpha = 5\%$	$\alpha = 10\%$
Outlier in largest group	-25 to 25	Bartlett's	3	3	7
		Proposed	6	11	12

5. Bahadur's Asymptotic Relative Efficiency - The two tests were compared using Bahadur's ARE. The proposed test was found to be more efficient in all the cases. e.g. for imbalanced case 3 with 4 groups, and equal group size = 6, Bahadur's ARE = 1.1828.

REFERENCES

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