ON THE SHEWHART’S OPERATIVE CHARACTERISTIC CURVE

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1. Introduction

To emphasize the Operating Characteristic (O.C.) Curve importance in Shewhart’s variables Control Chart, is the main purpose of this paper. Indeed the use of O.C. Curve leads to protection knowledge and thereby to a correct evaluation of sample size (n). It is not infrequent to find suggestions to use n=5; but on the interpretation of the corresponding O.C. we realize the risk we are really running if produced quality is bad ($P \geq P_1$) as we can later see. In synthesis we have a tool for sample size evaluation.

Showing the proposal of factory needs data pre-assignment and use in OC construction is another important purpose. Factory’s data are (Mittag-Rinne, 1993):

$L = \text{lower specification limit } \bar{x}$;
$U = \text{upper specification limit } s$;
$P_0 = \text{AQL = acceptable quality level}$;
$LTPD = \text{lot tolerance percent defective}$;
$\alpha = \text{producer’s risk = first type error probability}$;
$\beta = \text{consumer’s risk = second type error probability}$.

Having these data we are able to compute $\bar{x}$ and $s$, parameters to be used for control limits computation. This proposal is very interesting because it implies the first stage data collection cancellation and therefore some time and money saving.

2. Control limits computation

As it is well known Shewart’s variables control chart requires the collection of m samples n sized, randomly drawn from m groups N sized, produced by the machines to be controlled. These data are synthesised by means of $\bar{x}$ and $s$, general mean and general standard deviation.

The computed values are a measure of production process capability, particularly $s$ and they do not take into account designer’s needs (Giacalone, 1997).

Almost always production begins with a design where we find $L$ and $U$ for many parts to be produced. Now if the designer let us know the fraction of items he can accept less than $L$, let us say $L_P$, and the fraction he can accept over $U$, let us say $U_P$ ($L_P + U_P = P_0$), if the variable of interest is normally distributed, we are able to evaluate $z_L$ (normal deviate based on $L_P$) and $z_U$ (normal deviate based on $U_P$), afterwards we can compute:

$$\bar{x} = \left( L_U - U_L \right) / \left( Z_U - Z_L \right), \quad Z_L < 0$$
$$\sigma = \left( U - L \right) / \left( Z_U - Z_L \right)$$
Therefore control limits become:

\[
\begin{align*}
UCL_X &= \bar{X} + z_\alpha S/\sqrt{n}; \\
LCL_X &= \bar{X} - z_\alpha S/\sqrt{n}; \\
UCL_S &= S \cdot \sqrt{\chi^2_{\alpha}/(n-1)};
\end{align*}
\]

where \( z (<0) \) is the normal deviate, based on \( \alpha \) at the left of the normal distribution, \( z \) is the normal deviate, also based on \( \alpha \), but at the right of the normal distribution, \( \chi^2_{\alpha} \) is the particular value of the homonymous distribution leaving a probability of first error equal to \( \alpha \) at its right.

3. Conclusions

There is a recent handbook (Mittag-Rinne, 1993) bearing, on its cover page, the three dimensional plot of the Shewart’s power function. We remember that the last function is complementary to the O.C. and it is built considering the rejection probability.

Unfortunately there is no possibility interpretation because, for every sample size, there is areas of high and low slope. On the contrary a two dimension power function with argument \( P \) (quality) may be interpreted by comparison with \( 1-\beta \).

To reach this goal we follow Sheffe’s basic ideas. Rouzet (1957) suggested the use of \( P \) as the argument for the function but getting a family of functions, whilst Panizzon (1974) reduced the family to a simple function.

The methodology adopted has been a critical exam of existing references on the subject studied. The main result obtained is the organic collection of papers dealing with the computation of Shewart’s Control Chart acceptance probability. In our study in progress we are considering a reasonable definitive solution.

References


