A non-linear regression for physical growth of Japanese

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1. Introduction

Various studies on nonlinear growth curve of human physical growth have been developed since 1930’s. On the early stage of growth studies, simple models that include a few parameters were used. In order to describe human growth from birth to maturity with an asymptotic mathematical formula, some advanced models have been proposed. From a practical point of view, we need to choose a suitable asymptotic mathematical function as a growth curve according to the situation and the purpose of analysis. Here, we study three models from various points of view, particularly from a viewpoint of practical growth analysis.

2. Materials

We used a part of the Hiroshima Growth Study Sample consisting of 3,325 Japanese. When they matriculated in a college at Hiroshima, the growth records were collected retrospectively. They were born between 1964 and 1968 and all samples have an observation at birth. Almost all of them have one observation on each spring between the age of six and eighteen, but they were not measured frequently between the age of one and six. In order to study the characteristic on growth curves, we selected 115 boys and 112 girls who have at least five observations between age of one and six, and the time intervals of successive measurements do not exceed three years.

3. Growth model

The growth record of $i$-th individual is described as $(t_{ij}, h_{ij})$, where $j$ is the index for measurements ($j=1,\ldots,n_i$). We use a growth model such as $h_{ij}=f(t_{ij};\theta)+e_{ij}$ without repeated measurements. Here, $f(t;\theta)$ is a non-linear growth curve, $\theta$ is an unknown growth parameter vector for $i$-th individual, and $e_{ij}$ is a measurement error. We assume $E(e_{ij})=0$ and $V(e_{ij})=\sigma^2$. Various forms of growth functions are proposed for $f(t_{ij};\theta)$ in order to describe human growth from birth to maturity. Here, we deal with the following three models; triple logistic curve (Bock and Tissen, 1976)

$$f(t;\theta) = \left( \frac{1-D}{1+e^{-A_1(t-B_1)}} + \frac{D}{1+e^{-A_2(t-B_2)}} \right) + \frac{U-C}{1+e^{-A_3(t-B_3)}}$$

slightly modified Count-Gompertz curve (Kanefuji and Shohoji, 1990)

$$f(t;\theta) = \left( C + D(t+E \ln(t+F)) \left( 1-e^{-e^{-A(t-B)}} \right) \right) + U e^{-e^{-A(t-B)}}$$

and JPA2 curves (Jolicoeur et al., 1992).

$$f(t;\theta) = U \left( 1 - \frac{1}{1 + \left( (t+E)/b_1 \right)^v + \left( (t+E)/b_2 \right)^v + \left( (t+E)/b_3 \right)^v} \right)$$

We have successfully obtained the estimates for the samples using Marquardt method for nonlinear optimization.
4. Biological Parameters

For structural growth curves, we have an advantage that some characteristic points of growth curve can be defined mathematically. The local minima and maxima of growth velocity curve which is the derivative of growth curve with respect to age \( t \), play important role to characterize an individual growth. We call the values (age, height) of characteristics points of growth curve as biological parameters. Here, we concern three sets of biological parameters: 1) the point at minimum height velocity (MHV), 2) the point at peak height velocity (PHV) and 3) the point at growth completion (GC). For each point, the age, the height and the amount of growth velocity are concerned. For example, the age attaining MHV is referred as AMHV and the height at PHV is referred as HPHV. Conventionally, we call the height at GC as AH. The graphical representation for these biological parameters is shown in Figure 1.

5. Comparisons of growth curves

From a viewpoint of practical analysis, it is more important to select a mathematical growth curve with its characteristic, in addition of the statistical goodness of fit. Here, we will discuss above three growth models with the variances of estimators of growth parameters and biological parameters, the location of biological parameters, difficulty in estimation and so on.

The followings are some results. Numerically, triple logistics (TL) model and Jolicoeur et al. (JPA2) model have better fitness than modified Kanefuji and Shohoji (KS2) model. However, the optimization procedure is easily performed for KS2 model. There are slight but consistent differences among the estimates of biological parameters of these three growth models. In case that growth analysis may deal with the biological parameters, the characteristics of growth curve should be concerned. The estimates of biological parameters are the most stable for KS2 model; other models have relatively large variations. For a data set of which the length of interval between consecutive measurements is large, KS2 model is better than other two models because the variation of estimates of growth parameter is relatively small for such a data set.

REFERENCES


RÉSUMÉ

Nous traitons le grossissement individuel de la taille humaine en utilisant le model grossi non-linaire. De la point de vue pratique, il faut de choisir la fonction mathematique appropriée comme la courbe grossie selon la situation et le but analyse. Nous recherchons trois courbes grossis du les point de vue variées pour le choix du courbes grossis appropriés.