Stochastic Geometry and Statistics of Telecommunications

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1. Necessity of modelling

Optimisation of the network’s architecture is one of the principal ways for telecommunication operators to increase the effectiveness of their system. The strategic planning is based on an economical analysis of statistical data and aims to find an architecture that best meets the future service demand. The current planning methods require the complete knowledge of the network and the traffic together with a prediction for the future service demand. Therefore the corresponding optimisation programs use a very large number of parameters the values of which are not all known exactly, and due to their specificity and complexity can not be applied to other networks. Moreover, the complete description does not give a transparent functional relation between the performance characteristics and the descriptive parameters of the network’s topology.

Therefore development of reliable models possessing a few number of structuring parameters and taking into account spatial characteristics of the network opens many possibilities. Such models could be useful for preliminary studies of different development scenarios. They make possible to evaluate the evolution and productivity cost for many configurations and architectures (comprising the cost of competitors’ networks) using only a few main topological characteristics. They direct gathering and correct interpretation of the relevant statistical data.

2. Stochastic geometry approach

The approach developed in the framework of the research contract between INRIA and CNET consists in representing the current configuration of the system elements (cables, concentration points, stations of different hierarchical levels, ground antennae, etc.) as a realization of a spatial stochastic process (see, http://www.dmi.ens.fr/~mistral/sg/ and the references therein). The spatial variability of the system’s topology and the related performance characteristics of the network may now be described in terms of functionals of the corresponding random sets and so can be studied by probabilistic and statistical techniques (see [2]).

3. Architecture of distribution networks

Consider the basic concept of a service zone, i.e. the geographical area served by a station (a switch). If one takes a reasonable assumption that the network’s subscribers are served by the nearest station and that the whole area is covered by the network’s service then the zones form the Voronoi tessellation with stations being the nuclei set. In the basic model the latter is modelled by a homogeneous Poisson point process. Geometric characteristics of the service zones are thus related to the ones of the Poisson-Voronoi cells for which many analytical and simulations results are available (see, e. g. [9]). Let \( \Pi_0, \Pi_1 \) be stochastically independent Poisson processes representing the subscribers and the lower level stations, respectively. Important class of characteristics is related to
a typical zone $C_0(\Pi_1)$ mathematically described as the Voronoi cell constructed with respect to the process $\Pi_1 \cup \{0\}$ with the nucleus at the origin 0. Examples are provided by the functionals of the form

$$S_\beta = S_\beta(\Pi_0, \Pi_1) = \sum_{x \in \Pi_0} \| x \|^{\beta} \mathbb{1}\{ x \in C_0(\Pi_1) \}$$

that describe the number of subscribers $x_i$ in a typical zone $C_0$ if $\beta = 0$, the total length of connections in a typical cell for a star-type network shown in Figure 1 if $\beta = 1$, or the cost of these connections that takes into account the economy of the scale if $0 < \beta < 1$.

Paper [5] gives expressions for the first two moments and the tail decay of the Palm distribution of $S_\beta$ in terms of the intensities $\lambda_i$ of the processes $\Pi_i$ ($i = 0, 1$). These results can be used for statistical estimation of the parameter $\beta$ and hence the total cost of the system’s connections. For statistical analysis of the cost, the results on the rate of the Palm probability’s tail decay $P_1^0\{S_\beta > x\} \asymp \exp\{-\text{Const}(\lambda_0, \lambda_1) \cdot x^{2/(2+\beta)}\}$ may be useful, suggesting, in particular, a generalised Gamma distribution for $S_\beta$.

The basic model can be iterated to include stations of higher hierarchical levels. The number and levels of the intermediate stations carrying a call, that is its main cost factor, can be related in the framework of these hierarchical models to the distance between the calling parties as it was done in [1]. More elaborated models for the distribution level of a network that distinguish between media (cable) and environment (trench) cost were proposed and studied in [4].
4. Wireless networks

A specific feature of mobile systems concern the temporal evolution. Customers mobility leads to switches in the service disciplines as subscribers cross the base station zones (known as ‘hand-over’). Similar problems arise in satellite communications where the hand-over is mainly due to movement across the radio-horizon of satellites deployed on low Earth orbits. Since this strongly affects the cost and reliability of the service, the studying and optimisation of hand-over related characteristics is one of the primary goals of wireless telecommunications operators. In [3] we propose stochastic spatial-temporal models and give expressions for the hand-over intensity via such characteristics as the density of the base stations, density of the roads, customers’ mobility and the intensity of calls. Other interesting stochastic models for mobility are studied in [7].

5. From homogeneity to non-homogeneity (and back)

Most analytic results were obtained for the models based on homogeneous processes. Additional statistical studies are thus necessary to check if the homogeneous assumption is realistic. Analysis of a few cities in France shows that this could well be the case on the scale of Metropolitan Area Networks. The consideration needs to be restricted to an area where the population density could be assumed constant (like urban or residential areas).

On the national scale the homogeneity hypothesis should undoubtedly be rejected (at least for France). In this case change of the phase space methods could be useful. They are based on the fact that, given a non-constant function \( \phi(x) \) on a subset \( F \subset \mathbb{R}^2 \), there exists a transformation \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) such that the image \( \phi \circ T^{-1} \) of \( \phi \) on \( T(F) \) is a constant function (see, e. g. [1]).

Note that the stochastic independence of the processes, assumed in the models above, does not mean the parameters of their distributions are not related. Heuristics known to network planners and backed by theoretical results obtained in [8] state that the optimal density of stations should be proportional to a power of the demographical density. Therefore a reasonable generalisation of the basic model described in would be a model with \( \Pi_\alpha \), \( \pi_1 \) being independent Poisson processes with the intensity measures (densities) \( \phi(x) \) and \( \phi^\alpha(x) \), respectively, for some \( 0 < \alpha < 1 \). The above bijection \( T \) then transforms this model to a homogeneous one. The statistical results then could be interpreted as conditional ones given the value of the demographic density at the considered place. The distortion of the Voronoi cells by application of \( T \) in many cases could, perhaps, be tolerated.

6. Conclusion

We described some of the models based on stochastic geometry which have been used by France Telecom for different economical studies of networks and briefly commented on validity of their assumptions. The models are generic in the sense that they may be used for other types of cellular systems. The advantage of these models is that they reduce significantly the number of the structuring parameters to just a few parameters of the underlying stochastic system. This facilitates statistical and analytical analysis of the model and allows for the determination and correct interpretation of the main factors influencing the performance.
REFERENCES


FRENCH RÉSUMÉ

Nous discutons l’approche fondée sur des méthodes de la géométrie aléatoire à la modélisation de l’architecture de réseaux de télécommunications. Plusieurs résultats analytiques obtenus pour les modèles poissoniens et la méthode de changement de l’espace de phase permettent le traitement statistique adéquat des caractéristiques spatiales des réseaux et donnent souvent l’expression directe des indicateurs discriminants de la fonction de coût à un niveau macroscopique.