Testing Generalized Lorenz Dominance

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It is well known that the property of generalized Lorenz dominance (GLD) between two distributions is a scaled Lorenz dominance, where the scale factor is the ratio of the two finite positive means. It is convenient to test the GLD property between any pair of distributions because, the property is equivalent to the so-called second order stochastic dominance (SSD), whose statistical tests may be formulated via empirical distributions. The literature has provided many discussions about testing SSD. The aim of this study is to combine a few existing ideas to come up a modified test for SSD, hence a useful test for GLD.

Assume without loss of generality the two distributions are supported on a common interval of the non-negative real line. Like the test design of McFadden (1989), we will consider testing the null hypothesis

\[ H_0 : \int_0^x F(t) dt \geq \int_0^x G(t) dt \text{ for all } x \in [0, \infty), \]

against the alternative

\[ H_1 : \int_0^x F(t) dt < \int_0^x G(t) dt \text{ for some } x \in [0, \infty), \]

One expects to reject \( H_0 \) in favor of \( H_1 \), when \( X \) (\( \geq \text{d} F \)) is not second-order dominated by \( Y \) (\( \geq \text{d} G \)). Let \( X_1, \ldots, X_m \) and \( Y_1, \ldots, Y_n \) be two independent samples of sizes \( m \) and \( n \), from distributions \( F \) and \( G \), respectively. Let \( F_m \) and \( G_n \) denoted their empirical distributions. McFadden (1989) developed a test for (1) based on the statistic \( S_n = \max_x \sqrt{n} \int_0^x [F_n(t) - G_n(t)] dt \), where the sample sizes were assumed equal. However, Kaur, Prakasa Rao and Singh (1994) (henceforth KPRS) noted that the distribution of \( S_n \) does not have tractable analytical form and its asymptotic distribution under \( H_0 \) was not fully characterized. By reversing the null and alternative hypotheses in (1), KPRS constructed a test through the statistic \( Z_{m,n}(x) \), which is the sample mean difference \( \int_0^x [F_m(t) - G_n(t)] dt \), normalized by its sample standard deviation. They argued that based on the intersection-union principle, the test statistic \( Z_{m,n} = \inf_x Z_{m,n}(x) \) would achieve desirable testing performance.

In light of a simulation study, it is of interest to consider a modified test statistic based on that of KPRS, however under the testing framework of (1). In words, a test that combines the ideas of the two previous studies can be shown to enjoy standard asymptotic properties. The main theorem and a simulation study will be presented to illustrate the basic findings, but omitted here for brevity.

REFERENCES


SUMMARY

The notion of Lorenz dominance has played an important role in the econometric theory. Statistical tests for the relevant stochastic dominance properties have been much discussed in the literature. This study, motivated by two recent studies, considers a modified test for generalized Lorenz dominance and illustrates the test performance using a simulation study.