Simulated Asymptotic Least Squares Theory

Ramdan Dridi

London School of Economics and Political Science,
Houghton Street,
London WC2A 2AE United Kingdom
Email address: r.dridi@lse.ac.uk

Econometric models often lead nowadays to a complex formulation of either conditional probability distribution function (p.d.f. hereafter) of the endogenous variables given the exogenous and predetermined ones in the fully parametric setting or more generally to a complex formulation of the so-called estimating equations in the semiparametric setting. Optimizing behavior and optimizing subroutines are such that one cannot directly write the functional forms associated with a given parametrization. In this context, several approaches depending on the primitive setting and circumventing such difficulties have been introduced in the literature: the Simulated Pseudo Maximum Likelihood (SPML hereafter) by Laroque and Salanié (1989), the Simulated Method of Moments (SMM hereafter) by McFadden (1989), Pakes and Pollard (1989), Ingram and Lee (1991), Duffie and Singleton (1993), the Simulated Nonlinear Least Squares (SNLS) by Laffont, Ossard and Vuong (1995) and more recently the Indirect Inference (II hereafter) and the Efficient Method of Moments (EMM hereafter) respectively by Gouriéroux, Monfort and Renault (1993) and Gallant and Tauchen (1996). However, it turns out, on the one hand, that so far no general econometric theory enabling the unification of all the aforementioned methodologies has been proposed. On the other hand, there are now cases arising from the macroeconometric as well as the econometric literature where the sole application of such approaches does not fully exploit the information brought about by the available estimating equations. This occurs, for instance, when one has at his disposal overidentifying moment restrictions defining a set of instrumental parameters in conjunction with a fully specified parametric structural model. In this respect, we propose in this paper a general econometric theory referred to as the Simulated Asymptotic Least Squares (SALS hereafter). It is shown that this approach provides a unifying theory for simulation-based or more generally “approximand” based inference methods and nests the SPML, the SMM both in parametric and semiparametric settings, the II, the EMM and the SNLS approaches. It can indeed be regarded, on the one hand, as a simulated or approximand extension of the earlier ALS theory introduced by Gouriéroux, Monfort and Trognon (1985) to the case where the estimating equations are intractable but can be approximated in some sense either by simulations or more generally by approximand methods such as for instance the quadrature-based methods or Marcet parametrized expectations type procedures. But on the other hand, it can also be regarded as a generalization of the ALS theory to the case where the number of estimating equations \( r \) (say) is bigger than the number of auxiliary parameters \( q \) (say), that is \( r > q \). Indeed, we stress in this paper that while in Gouriéroux, Monfort and Trognon (1985) the ALS theory is developed in the particular case where \( r < q \), there are now numerous examples where \( r > q \). We provide here a general study dealing with both issues. In this respect we think that the (S)ALS theory should enjoy some renewal especially in light of the now increasing literature in macroeconometrics and more generally in econometrics often leading to restrictions or estimating equations that are poorly handled by the common simulation-based methods. Besides, it enables the exact and precise characterization of what is now abusively referred to as the “matching” in the approximand methods through the use of the estimating equations. Each existing methodology (SPML, SMM, II, EMM, and SNLS) is thus characterized by particular estimating equations. The paper is organized as follows. We first recall in section 2 and prove under weaker stochastic equicontinuity conditions the available results from the II. In section 3, we develop the Generalized Indirect Inference (GII) seen as a particular and introductory illustration to the SALS
theory. While analyzing the efficiency gains brought about by additional constraints (such as Euler conditions) on the instrumental criterion in the II framework, we show that Hansen (1982) theory of efficient overidentified moments estimation is no longer available here and characterize the new weighting matrix for performing a more efficient indirect estimation about the structural parameters of interest. In section 4, we develop the SALS theory and provide the general efficiency study in the SALS framework. We are thus led to introduce a new notion of **Efficiency Bounds in Direction**. Section 5 proposes a battery of generalized global specification tests extending the previous existing ones. We state some concluding remarks in section 6.¹


**RÉSUMÉ**

La sophistication des modèles a conduit progressivement à introduire des méthodes simulées ou approchées, il faut aussi noter d’une part qu’il n’existe pas de cadre suffisamment général pour unifier toutes les approches et méthodes existantes. D’autre part, il existe de nos jours des cas pour lesquels ces approches n’utilisent pas de façon optimale ou plus précisément de manière efficace toute l’information disponible. Nous proposons, par conséquent, d’introduire un cadre général nommé Moindres Carrés Asymptotiques Simulés (SALS) qui fournit une réponse aux questions précédentes.

¹ Because of place restrictions, we do not present the whole theory but it is available from Dridi (1999) “Simulated Asymptotic Least Squares Theory” mimeo LSE.