Calibration of Structural Models By Semiparametric Indirect Inference

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There is a fairly general agreement about the two main goals of Econometrics, as defined by Christ (1966): « the production of quantitative economic statements that either explain the behavior of variables that we have already seen, or forecast (i.e. predict) behavior that we have not yet seen, or both ». In any case, this activity relies not only upon empirical facts but also upon theory, which produces explanation or forecasting. But, the best way to reach these goals is a matter of controversy since the very beginning of Econometrics in the thirties. Moreover, with the advent of real-business-cycle models in the eighties, the so-called « calibration methods » have been presented as an alternative to « orthodox statistics », because the economic theory is considered as « ahead of measurement » as argued for instance by Prescott (1985).

This is the reason why some statisticians like Pagan (1995) are led to make fun of calibrators by considering them « very close to blaming the data if the calibrator’s model fails to fit ». But, to some extent, new classical macroeconomics pleads guilty concerning this accusation by answering that « the interesting question is surely not whether the real-business-cycle model can be accepted as ‘true’ when nested within some broader class of models. Of course the model is not ‘true’ (...). We know from the onset in an enterprise like this that what will emerge-at best-is a workable approximation that is useful in answering a limited set of questions. » (Lucas 1987).

Amazingly, this regained freedom of quantitative economic theory (as proposed by calibrators) with respect to statistical orthodoxy is acknowledged by both its detractors and its proponents. While the formers consider that this makes questionable the credibility of calibrators « computational experiments », the latter claim this freedom by using « the mantel of Frisch » (Hoover (1995)) to argue that econometrics is not coextensive with estimation and testing, that is with orthodox statistics. More precisely, Kydland and Prescott (1991) claim that calibration is also econometrics by referring to Frisch (1970) review of the state of econometrics: ‘In this review he discusses what he considers to be « econometric analysis of the genuine kind » (p163), and gives four examples of such analysis. None of these examples involves the estimation and statistical testing of some model. None involves an attempt to discover some true relationship. »

Such an « endorsement of calibration as an alternative to estimation » (Hansen and Heckman (1996)) leads one to the conclusion that « the new classical macroeconomics is now divided between calibrators and estimators » (Hoover (1995)). Actually, we share with Hansen and Heckman (1996) the opinion that ‘the construction of such artificial distinctions is counterproductive » and the main goal of this paper is to try to delineate a close methodology which could gather both the advantages of the inferential approach (estimation and specification testing) and also the advantages of the calibration approach, that correspond, in our opinion, to consistent
estimation of some structural parameters of interest and robust predictions despite misspecifications in the structural model used as a simulator. In this respect, it is true that we should not be obsessed by « estimation and statistical testing of some model », viewed as « an attempt to discover some true relationship » but we consider that the calibrationist practice can be « fruitfully posed as econometric estimation and testing problems » of something else than a « true unknown model ».

Actually, the model is nothing but « a workable approximation that is useful in answering a limited set of questions » (Lucas 1987, op cit) and these questions define the structural parameters, which are the focus of interest of the inferential approach. In order to be more precise on this latter point and to introduce the main results of the paper, let us use the famous Mehra and Prescott (1985) paper as a template of the calibrationist methodology. This paper addresses the question whether the large differential between the average return on equity and average risk free interest rate can be accounted for by models neglecting any friction in the Arrow-Debreu set up. The simple statement of this question defines, on the one hand the structural parameters of interest and on the other hand the instrumental parameters through which the empirical evidence is summarized. More precisely, the benchmark model is the consumption based CAPM with a single representative agent characterized by a time separable expected utility function and a constant relative risk aversion. In other words the structural parameters of interest for Mehra and Prescott’s question are the two taste parameters of the representative agent: \( \theta_1 = (\alpha, \gamma) \).

Of course, this way of economically defining the structural parameters of interest is tightly linked to the economic setting the modeller has in mind and might be reducing since for instance while \( \gamma \) represents the psychological discount factor \( \alpha \) represents both relative risk aversion and inverse of the elasticity of intertemporal substitution. This implicitly assumes that this reduction has no incidence on the answer to the aforementioned question of interest (the so-called « equity premium puzzle »). Moreover, in this approach the structural model is empirically assessed only through its ability to reproduce the stylized facts namely the observed high value of the equity premium. In our statistical framework, these stylized facts are referred to as the set of instrumental parameter denoted \( \beta \) which are easily estimated by their empirical counterpart.

Actually one the most difficult issue for a close statement of the calibration methodology is that, to assess the goodness of fit, one has to rely on additional assumptions which are neither given by any economic theory nor by any statistical procedure. These additional assumptions may require the specification of additional parameters \( \theta_2 \) possibly of infinite dimension. In Mehra and Prescott (1985), these parameters \( \theta_2 \) define the endowment process that is the Markov chain assumed to govern the gross rate of dividend payments. More generally, the vector \( \theta \) of structural parameters is split into two parts \( \theta_1 \) and \( \theta_2 \) where \( \theta_1 \) gathers the characteristics of interest and \( \theta_2 \) corresponds to nuisance parameters, which are needed for the statistical procedure described below. The most usual case is the one where \( \theta_1 \) is related to preference specifications (taste parameters) and \( \theta_2 \) describes environmental characteristics (technology parameters). In any case, the main role of these nuisance parameters \( \theta_2 \) consists in indexing a binding function between the structural parameters of interest \( \theta_1 \) and the instrumental parameters \( \beta : \beta = \tilde{\beta}(\theta_1, \theta_2) \).

The specific feature of the calibration methodology with respect to more standard statistical inference appears precisely at this stage: since our goal is to ask whether, given the technology, there exist taste parameters capable of matching the returns data, this, according to Cechetti, Lam and Mark (1993) « dictates that we proceed in two steps, first estimating the parameters of the endowment process, and then computing a confidence bound for the taste parameters \( \gamma \) and \( \alpha " \). With respect to more orthodox econometrics, this two steps procedure may arouse at least two types of criticism:

First, even though the only parameters of interest are the taste ones \( \theta_1 \) one get in general more accurate estimator by a joint, possibly efficient, estimation of \( \theta = (\theta_1, \theta_2) \).
Second, even ignoring the efficiency issue, it is somewhat questionable with regard to consistent estimation to focus on taste parameters while the technology corresponds to a caricature of reality. Nobody may believe that the endowment process is conformable with a two states Markov chain and this misspecification presumably contaminates the estimation of the parameters of interest.

In our opinion, a garbled answer to this apparent puzzle would consist in claiming that this procedure should not be regarded as an econometric one attempting to consistently estimate the parameters of interest. In this respect, we share Hansen and Heckman (1996) point of view: « the distinction drawn between calibrating and estimating the parameters of a model is artificial at best ». Actually, the core principle of the calibration approach as illustrated in our benchmark setting of equity premium puzzle consists in concluding that the structural model is rejected on grounds of « computational experiments » leading to unlikely values of the parameters of interest. Namely, Mehra and Prescott (1985) argue that computed value of the discount factor and the relative risk aversion parameter outside their commonly acknowledged range \((0 < \gamma < 1, 0 \leq \alpha \leq 10)\) proves the misspecification of the structural model. How could they maintain such an argument if they did not think that these « computed values » are consistent estimators of something, which makes sense? Therefore we think that calibration should also be interpreted in terms of consistent estimation of the parameters of interest, even though this issue is addressed in a non-standard way in several respects:

First, as explained above, it is often addressed in a negative way. The model is rejected because the estimators of its alleged parameters are obviously inconsistent. Second, consistency is the only focus of interest. Efficiency is irrelevant in this setting since the calibration exercises gather a huge amount of historical information such as series of asset returns over the whole last century in such a way that the efficient use of the information is not an issue at all. Third, calibrators are fully aware that consistency might also fail for « bad reasons » like the misspecification of the technology. Indeed, fully cautious in that, they advocate calibration as a search for sensible values of the nuisance parameters \(\theta_2\).

The main goal of this paper is to statistically analyze into further details the latter point. The starting point of this paper is that the required statistical methodology is very close to the so-called indirect inference methodology (II) as recently introduced by Smith (1993), Gouriéroux, Monfort and Renault (1993) and Gallant and Tauchen (1996). But the focus of our interest here is an inference methodology which is not only indirect (we recover a consistent estimator of the structural parameters \(\theta\) from a preliminary estimation \(\hat{\beta}\) of some instrumental parameters \(\beta\) thanks to a (simulated) binding function \(\beta = \tilde{\beta}(\theta)\)) but also semiparametric since the binding function \(\tilde{\beta}(\hat{\theta}_2)\) is indexed by a hypothetical value \(\tilde{\theta}_2\) of \(\theta_2\) to recover an estimate \(\hat{\theta}_1\) of the parameters of interest \(\theta_1\).

In order to address the issue on consistently estimating the true unknown value \(\theta_1^*\) through a semiparametric indirect inference (SII), we introduce the notion of partial encompassing. The main difference with the older notion of encompassing as proposed by Mizon and Richard (1986), or for a simulated version by Dhaene, Gouriéroux and Scaillet (1998) is that the emphasis is led on the pseudo-true value of interest \(\left(\theta_1^*, \theta_2^*\right)\). Moreover, in this framework and when required by the partial encompassing property, some of the nuisance parameters \(\theta_{22}\) (say) are not estimated within the first step SII but in a second step introducing some general simulation-based loss function. This formalization enables us to derive the asymptotic probability distributions of the SII estimators but also Wald Encompassing Tests (WET) as well as Hausman type tests. The main message of the paper is that the semiparametric indirect inference methodology is a comprehensive statistical framework for calibration: since we know that the structural model is misspecified but we really need it for the interpretation of some structural parameters, we try to estimate it only through well-chosen characteristics which are conformable to the main purpose of the model. The underlying
philosophy is that some elements of truth involved in the model should be caught by matching only some well-chosen moments and not a too large set of moments prompted by an automatic statistical process. Otherwise, we might get an inconsistent estimator of the parameters of interest as well as unreliable predictions, due to a contamination in dimensions where the model may do miserably. This is nothing but the often mentioned « discipline of the calibration method » which « comes from the paucity of free parameters » (see Hoover (1995).

References


French Résumé

On développe dans cet article une généralisation de l’inférence indirecte à un cadre semiparamétrique qui permet de rendre compte de l’approche dite du calibrage, dans laquelle des modèles économétriques structurels sont étudiés seulement au regard de leur capacité à reproduire certains moments bien choisis. On souligne que, en dépit d’un manque de formalisation statistique, la méthodologie controversée du calibrage est pertinente pour le statisticien confronté à des modèles structurels issus de la théorie économique et qui ne peuvent fournir qu’une approximation statistique imparfaite des dynamiques réelles. Pour produire une théorie de l’estimation robuste en dépit des erreurs de spécification inhérentes à ce contexte on introduit une notion d’ « enveloppement partiel » qui s’appuie sur un concept de pseudo-valeur vraie d’intérêt, rendant compte des a-priori structurels a-statistiques. On fournit une théorie asymptotique des estimateurs de ces pseudo-valeurs vraies ainsi que des tests d’enveloppement associés.