Choosing Rotation Patterns for Trend Estimation

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1. Introduction

Repeated surveys provide time series that enable trends in the variable of interest to be assessed. Most repeated surveys use rotation patterns which are special cases of the class of \( a-b-a(e) \) designs in which each selected unit is included for \( a \) consecutive periods, removed for \( b \) periods and then included again for a further \( a \) periods. The pattern is repeated so that units are included for a total of \( c \) occasions. Setting \( b = 0 \) gives an in-for-\( c \) rotation pattern.

The sample overlap associated with a rotations pattern induces a correlation structure in the sampling error of the original series and the resulting seasonally adjusted trend series. McLaren and Steel (1997) considered the impact of different rotation patterns on the sampling variance of trend estimates obtained by applying Henderson (1918) moving averages (HMAs) to seasonally adjusted estimates obtained from X11 and X11ARIMA. Considerable gains could be obtained by using rotation patterns with no monthly overlap such as 1-2-1(8). The HMAs were originally derived assuming that the original series has an independent error structure. McLaren and Steel (1998) considered more general trend filters that take into account the correlation structure of the series and showed that the benefits of rotation designs such as 1-2-1(8) still applied. These results assumed that the trend estimates are obtained by applying a filter to a series of aggregate estimates. Here we develop trend filters and consider the effect of different rotation patterns on the properties of trend estimates when elementary estimates produced from each rotation group are available.

2. Trend Estimation Using Aggregate Estimates

Let \( y_{tg} \) be the elementary estimate for time \( t \) in rotation group \( g \), where \( t = 1, \ldots, N \) and \( g = 1, \ldots, G \). Write \( y_g = (y_{tg}, \ldots, y_{tN_g})' \) as the vector of estimates for rotation group \( g \) and the complete vector of elementary estimates and \( y = (y_1, \ldots, y_G)' \). The value of the characteristic of interest in the population at time \( t \) is \( Y_t \) and \( Y = (Y_1, \ldots, Y_N)' \). We initially consider trend estimates based on aggregate estimates of \( Y \) of the form

\[
\hat{Y} = Ay
\]

where \( A \) is an \( N \times NG \) matrix. We assume that the elementary estimates are design unbiased, so \( E(y|Y) = X_N Y \) where \( X_N = 1_G \otimes I_N \). Let \( V(y|Y) = \nu \nu' \). Assuming that we know \( \nu \nu' \), the best linear unbiased estimate (BLUE) is obtained by using \( A = (X_N' \nu \nu' X_N)^{-1} X_N' \nu \nu' \). Yansaneh and Fuller (1998) noted that computation of the BLUE becomes complicated as the number of periods increase. They developed a recursive regression estimator which avoids the complexity of the BLUE approach. A way to reduce the computational burden associated with the BLUE approach is to restrict its calculation to the last \( m \) periods. Yansaneh and Fuller (1998) suggest using \( m = 16 \) is almost as efficient as the restricted regression estimator.
and $m = 36$ gave virtually the same efficiency as the recursive regression estimator.

McLaren and Steel (1998) developed trend filters for repeated surveys based on an approach due to Gray and Thomson (1996). Let $\hat{Y}_t$ be a design unbiased estimate of $Y_t$, then

$$\hat{Y}_t - Y_t + e_t$$

where $e_t$ is the sampling error associated with $Y_t$ and $E[e_t | Y_t] = 0$. It is assumed that $Y_t = T_t + \eta_t$ where $T_t$ is a trend-cycle component and $\eta_t$ is an irregular component. No seasonal factors are included although the methods can be extended to seasonally adjusted series. Consider a window of length $n = 2r + 1$ centered at time period $t$ and assume data are available to $t + r$. It is assumed that within the window the trend follows a local dynamic model of order $p$

$$T_{t+i} = \sum_{j=0}^{p} \beta_j i^j + \varepsilon_i$$

where $\beta_j$ are parameters specific to the window and $\varepsilon_t$ is a correlated process with zero mean, uncorrelated with $\varepsilon_t = \eta_t + e_t$. Let $\mathbf{e}_t$, $\eta_t$, $\varepsilon_t$, $\varepsilon_t$ be vectors of length $n$, centered at $t$ containing the elements $e_{t+i}$, $\eta_{t+i}$, $\varepsilon_{t+i}$ and $\varepsilon_{t+i}$ respectively with $-r \leq i \leq r$. The associated $n \times n$ covariance matrices are $V_e$, $V_\eta$, $V_{\varepsilon} = V_e + V_\eta$ and $V_{\varepsilon}$. It is assumed that the error series is weakly stationary.

For an estimate $\hat{T}_t$ of the trend, fidelity and smoothness are defined as

$$F = E[(\hat{T}_t - T_t)^2] \quad S = E[(\Delta^{p+1}\hat{T}_t)^2]$$

Gray and Thomson (1996) suggested combining these criteria into a single criterion

$$Q = \theta F + (1 - \theta)S \quad 0 \leq \theta \leq 1$$

Consider the trend estimate for the central point, $t$, of the moving average window

$$\hat{T}_t = \sum_{i=-r}^{r} w_i \hat{Y}_{t+i}$$

Let $w = (w_r, \ldots, w_r)$ be the weight vector. Define $c = (1, 0, \ldots, 0)^T_{(p+1)}$

$$C = \begin{pmatrix}
1 
& -r & \cdots & (-r)^p \\
1 & -r + 1 & \cdots & (-r + 1)^p \\
\vdots & \vdots & \vdots & \vdots \\
1 & r - 1 & \cdots & (r - 1)^p \\
1 & r & \cdots & (r)^p \\
\end{pmatrix}_{n \times (p+1)}$$

$$\Omega = Var(\varepsilon_t - 1\varepsilon_t)$$

$$V_{e}^{(p+1)} = Var(\Delta^{p+1}\varepsilon_t) = D^{(p+1)}V_{e}D^{(p+1)'},$$

where $D^{(p+1)}$ is a $n \times (n + p + 1)$ matrix in which each row contains the coefficients of the process $\Delta^{p+1}\varepsilon_t$ and the length of the series is extended to include $n + p + 1$ values. Also define

$$\Gamma = D^{(p+1)}V_{e}D^{(p+1)'}$$
The matrices multiplied by $D^{(p+1)}$ are extended to be of size $(n + p + 1) \times (n + p + 1)$. Set

$$E = \theta(V_t + \Omega) + (1 - \theta)(V_t^{(p+1)} + \Gamma)$$

then $Q$ is minimised subject to $E[\hat{T}_i - T_i] = 0$ by using

$$w = E^{-1}C(C'E^{-1}C)^{-1}c$$

3. Trend Estimation Using Rotation Group Estimates

Consider applying a trend filter of length $n$ to the rotation group estimates to obtain

$$R\hat{T}_i = \sum_{i=-r}^{r} \sum_{g=1}^{G} \nu_{i,g} y_{t+i,g}$$

Let $w_g = (w_{rg}, \ldots, w_{rg})'$ and $Rw = (w'_1, \ldots, w'_G)'$ then

$$R\hat{T}_i - \sum_{g=1}^{G} w'_g y_{tg} = Rw'Ry$$

where $y_{tg}$ is the vector of length $n = 2r + 1$ of estimates for rotation group $g$ centered at period $t$ and $Ry = (y'_1, \ldots, y'_G)'$. Write

$$y_{tg} = Y_t + \epsilon_{tg}$$

where $\epsilon_{tg}$ is the sampling error at time $t$ for rotation group $g$.

Minimising $Q$ subject to the condition $E(R\hat{T}_i - T_i) = 0$ results in

$$Rw = RE^{-1}X_nC(C'X'_nRE^{-1}X_nC)^{-1}c$$

where $RE$ is a matrix determined by the correlation structure of the rotation group sampling errors. The trend estimate obtained directly from the rotation group estimates can be expressed in the form of a trend estimate based on an aggregate estimate.


As an application we considered choosing a rotation pattern for the Australian Monthly Labour Force Survey. We used the series of proportions of the number of persons employed. McLaren and Steel (1998) described a model for the correlation structure of the sampling errors for this series for different rotation patterns, based on correlation estimates in Bell (1998). We compared a number of commonly used rotation patterns using standard HMA’s and trend filters that account for the correlation structure. Fidelity and smoothness were calculated when the series used by the filters is the aggregate series obtained by simple averaging of the rotation group estimates, when BLUE estimates are used ($m=7,13, 37$) and when the rotation group estimates were used directly. The results showed that using rotation patterns such as 1-2-1(8) was better than using those with appreciable monthly overlap no matter what filtering approach was used. Moreover, for a 1-2-1(8) design use of the standard HIMA was as effective as use of more complex filters. If a rotation pattern with appreciable monthly overlap is used, then use of a BLUE estimates can have beneficial effects for trend estimation although the gains are less
than those obtained for the sampling error and using a standard HMA is as effective as more complex filters. Using trend estimates calculated directly from the rotation group estimates did not lead to any practical gains over those obtained from a series of BLUE estimate.

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REFERENCES


RESUME

Nous considérons l’effet de différentes configurations de rotation sur des évaluations de tendance obtenues en utilisant les filtres linéaires qui expliquent la structure de corrélation induite par la configuration de rotation. Différents filtres sont développés selon si l’analyse de tendance est basée sur le groupe élémentaire de rotation estimé ou des évaluations obtenues en agrégeant les évaluations de groupe de rotation. Les résultats suggèrent que l’approche commune d’avoir la superposition élevée d’échantillon entre les mois consécutifs ne soit pas la meilleure approche pour estimer des tendances.