Constant utility index is the same than Divisia’s index. Suggestion for a new index using another proxy.

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1. Equality between Divisia and Constant Utility Index.

P, q, k, t, U, y are prices, quantities, article, time between 0 and 1, utility function and consumer spending. The study deals with price index numbers.

Divisia index is (1) 
\[ I_D = \exp \left( \int_0^1 \frac{\sum_k (q_k(t))^* \frac{d}{dt} (P_k(t))}{\sum_k q_k(t)^* P_k(t)} \cdot dt \right). \]

Constant utility index (CUI) comes from : (2) \[ \max \ U(q) \quad s.t. \quad y = \sum p_i^* q_i. \] It is also called cost of living index. With an homogeneous utility function of degree 1 the curves of utility are homothetic and there is only one constant utility index (W.E. Diewert 1976).

from (2), the Lagrangien (3) \[ \frac{\partial}{\partial q_i} U(q) = \lambda^* p_i \] and property of homogeneous it is possible to show that (4) \[ \lambda^* y = \sum \lambda^* p_i^* q_i = \sum \frac{\partial}{\partial q}(q)^* q_i = U(q). \] Then, constant utility index is:

\[ I_{uc} = \frac{y(p(0);U(q(0)))}{y(p(1);U(q(1)))} \cdot \exp \left( - \int_0^1 \frac{d}{dt} \frac{\lambda(t)}{\lambda(1)} \cdot dt \right) \]

because of (4) and the passage from 0 to 1 is continuous. Some work using (3) and (4) gives (6) \[ \frac{d}{dt} \lambda(t) = - \frac{U(q)'}{y^2} \left( \sum \frac{dp_i^*}{dt} q_i \right). \]

Then : (7) \[ \frac{d}{dt} \frac{\lambda(t)}{\lambda(1)} = \frac{1}{y} \left( \sum \frac{dp_i^*}{dt} q_i \right) = - \frac{\sum dp_i^* q_i}{\sum q_i^* p_i}. \]

Therefore with (1) and (5) \[ I_D = I_{uc}^D = I_{uc}. \] QED


(8) \[ q_k(t) = q_k(0) \] gives Laspeyres’s index. (9) \[ q_k(t) = q_k(1) \] gives Paasche’s index.

(10) \[ q_k(t) = \frac{q_k(0) + q_k(1)}{2} \] first and in each term \[ q_k(t) = q_k(0) \] and \[ q_k(t) = q_k(1) \] gives Fisher’s index.

(11) \[ \omega_k(t) = \frac{q_k(t)^* P_k(t)}{\sum_{h} q_h(t)^* P_h(t)} = \omega_k(0) \] gives the geometric mean \[ I_g = \prod_k \left( \frac{P_k(1)}{P_k(0)} \right)^{\omega_k(0)} \]
3. The author proposal proxy. Instead of approaching the value or the quantities by a constant or an average, let us try to express the value by a polynomial of the price of degree N.

\[
\omega_k(t) = \frac{\omega_k(0) + \omega_k(1)}{2} \implies I^0_{tc} = \prod_k \left( \frac{P_k(1)}{P_k(0)} \right)^{\frac{\omega_k(0) + \omega_k(1)}{2}} = I_T, \text{ Törnqvist’s index.}
\]

\[
\omega(t)_k = \sum_{i=0}^{n} \alpha_{i,k} \cdot (p_k(t)) \implies I_n = \prod_k \left( \frac{P_k(1)}{P_k(0)} \right)^{\alpha_{i,k}} \cdot \prod_{i,k} \left[ \exp(p_k(1) - p_k(t)) \right]^{\alpha_{i,k}}.
\]

The author supports \( I_1 \). (It is notable that \( I_0 \) is the Törnqvist’s index.)

\[
I_k = \prod_k I_{1,k}; \quad (15) \quad I_{1,k} = \left( \frac{p_k(1)}{p_k(0)} \right)^{\omega_k(0) + \omega_k(1) - \omega_k(t)} \cdot \left[ \exp(\omega_k(1) - \omega_k(t)) \right]
\]

The index \( I_{1,k} \) is prolonged by continuity by (16) \( p_k(1) = p_k(0) \Rightarrow I_{1,k} = 1 \)

\[
p_k(1) = 0 \Rightarrow I_{1,k} = \exp(-\omega_k(0)) \text{ and } (18) \quad p_k(0) = 0 \Rightarrow I_{1,k} = \exp(\omega_k(1))
\]

The implicit value-price elasticity for \( I_1 \) (20) \( \frac{e_{wp}}{p_{1}}(t) = \frac{\omega_k(t) - \omega_k(0)}{p_k(1) - p_k(0)} \) is rather good.

3. Simulation made with weekly observations of prices and quantities of coffee sold during three years in France. (survey by Nielsen society, calculation by J. Pougnard, INSEE).

<table>
<thead>
<tr>
<th>method/index</th>
<th>Laspeyres</th>
<th>Paasche</th>
<th>Fisher</th>
<th>Geom.</th>
<th>Törnqvist</th>
<th>( I_1 )</th>
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</thead>
<tbody>
<tr>
<td>monthly chained calculations</td>
<td>150.402</td>
<td>124.805</td>
<td>137.007</td>
<td>146.299</td>
<td>137.219</td>
<td>137.148</td>
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<tr>
<td>monthly chained calculation with a price temporarily null.</td>
<td>151.721</td>
<td>123.410</td>
<td>136.835</td>
<td>0</td>
<td>indefinite</td>
<td>136.950</td>
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</tbody>
</table>

\( I_1 \) and Fisher’s index stay close. The author prefers \( I_1 \) because the proxy of a curve by a continuous series of segments has a direct meaning and gives sense to the value-price elasticity.

REFERENCES


FRENCH RÉSUMÉ

Cette étude montre l’égalité entre l’indice de Divisia et l’indice à utilité constante. Une nouvelle famille d’indices est proposée avec une approximation de la part de la valeur de la consommation par une expression polynomiale des prix. Au degré 1 l’élasticité valeur-prix implicite est mieux approchée qu’avec les indices courants. Une simulation numérique montre un comportement satisfaisant, proche de l’indice de Fisher, surtout face à des prix qui s’annulent temporairement.